Multiplication Strategies and Algorithms

Repeated Addition

What is it?

Students will add one factor the amount of times as the other factor. The product is the sum of that equation.

When do you use it?

This strategy is helpful with one or two digit factors.

Example: 3 x 16

```
3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 =
  V     V     V     V     V     V     V     V
  6     6     6     6     6     6     6     6
     12          12          12          12

          24             +          24          = 48
```

or

16 + 16 + 16 =

32 + 16 = 48

Milestones:

If the students can consistently use repeated addition, they are ready for equal groups.

Equal Groups

What is it?

Students will put the same number in each group and find the total.

When do you use it?

Students will use this strategy for finding products of beginning multiplication problems.

Example:

There are three spiders. Each spider has 8 legs. There are 24 legs total.

Milestones:

If the students can consistently group numbers into equal groups, they are ready for arrays.

Arrays

What is it?

Students group objects in rows of equal groups. An array is named in the following manner: # rows x # columns.
Although multiplication is commutative, in an array situation, a 1 x 10 array is not the same picture as a 10 x 1 array.

When do you use it?

Students will use this strategy to find products of beginning multiplication problems.

Example:

There are 4 different ways to make an array that has 10 objects.

Milestones:

If students can use an array to quickly find a product, they are ready to work with area models.

Area Model

What is it?

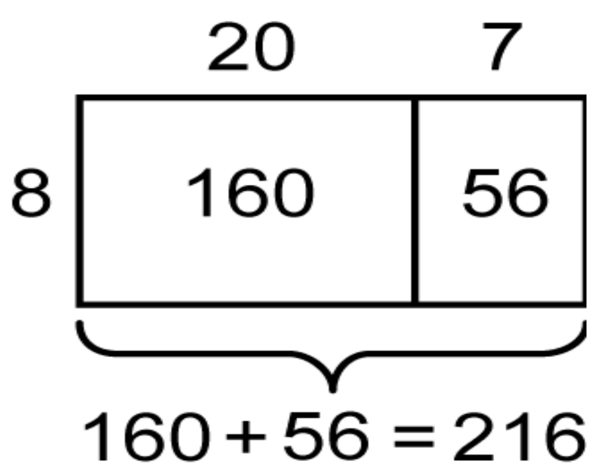
Students will use a proportionate visual model to represent breaking the factors apart to show that the length and width of a rectangle represent factors, and the area of the rectangle represents their product.

When do you use it?

Students will use their understanding of area in this strategy to find products of multiplication problems, where the products are easily decomposed.

Example:

If I know 8x20=160,and 8x7=56 then I can put those two products together to see that 160+56=216 and 8x27=216.



Milestones:

If students can use an area model to find a product, they are ready to work with area extended facts.

Extended Multiplication Facts

What is it?

Students will use a basic fact to help compute products of larger numbers.

When do you use it?

Students will use this strategy to find products of multiplication problems in which one or both factors are multiples of ten.

Example:

If I know3x8=24,then I can determine that 3x80=240 because 80 is the same as 8 tens, and 24 tens is the same as 240.

Milestones:

If students can consistently apply basic facts to find extended facts, they are ready to move on to partial products, including multiplying by place values.

By Place Value

What is it?

Students break apart a two-digit factor into tens and ones. Students then break the tens value into individual tens units. Next, students multiply the tens units by the other factor and the ones by the other factor. All partial products are then added together.

When do you use it?

This can be used to solve a one digit by two or more digit multiplication problem.

Example: 24 x 6

(24 = 20 + 4)

(20 = 10 + 10)

10 X 6 = 60

10 X 6 = 60

4 X 6 = 24

120

144

Partial Products

What is it?

Students break apart the factors into values (for example tens and ones) and multiply each part (value) of one factor by each (part) value of the other.

When do you use it?

This strategy can be used to solve one digit by two digit or larger multiplication problems. The entire Partials Algorithm is built on the premise of children thinking of numbers in pieces. We begin with children looking at problems horizontally so that they are more concerned with decomposing the number into its pieces than a procedure for solving the problem.

Example: 54 x 6

(54 = 50 + 4)

6 X 50 = 300
6 X 4 = 24
→ 324

or

	50	4
6	300	24

300 + 24 = 324

Example: 22 x 43

(22 = 20 + 2) (43 = 40 + 3)

20 X 40 = 800
20 X 3 = 60
→ 860

2 X 40 = 80
2 X 3 = 6
→ 86

860 + 86 → 946

Partitioning

What is it?

Students break apart the factors into parts (for example halves) and solve the easier problems. Then, students add the partial products together.

When do you use it?

This strategy can be used to solve one digit by two digit (or more) multiplication problems.

Example: 28 x 6

25 X 3 = 75

So, 25 X 6 = 150

3 X 6 = 18

150 + 18 → 168

Milestones:

If students can give a ballpark estimate of the product and understands place value and the distributive property, they are ready to move on to the traditional algorithm. Once children understand the process behind multiplying numbers in parts, it is time to transition them from horizontal problems to vertical problems.

Multiplication Notes

Alternative multiplication algorithms allow students to look at the values that make up numbers. Once students are able to multiply numbers in parts (values), they can begin vertical multiplication using alternative algorithms and then move on to the traditional algorithm.

Example: 47 x 6

$$
\begin{array}{ll}
 & 47 \\
 & \underline{\times 6} \\
6 \times 7 & 42 \\
40 \times 6 & \underline{+240} \\
 & 282
\end{array}
$$

Example: 37 x 23

$$
\begin{array}{ll}
 & 37 \\
 & \underline{\times 23} \\
3 \times 7 & 21 \\
30 \times 3 & 90 \\
20 \times 7 & 140 \\
20 \times 30 & +\underline{600} \\
 & 851
\end{array}
$$

Lattice Method

What is it?

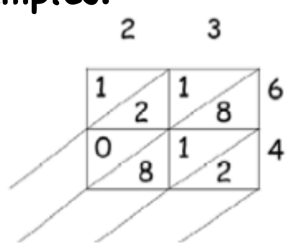
The factors are written on the outside of a lattice framework with diagonals drawn in. The products of each digit are written into the boxes and then added according to place value.

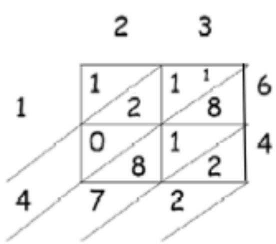
The diagonals of the lattice represent the traditional place- value columns.

When do you use it?

Students effectively use this algorithm after they have a thorough understanding of what happens with the distributive property and the

Examples:





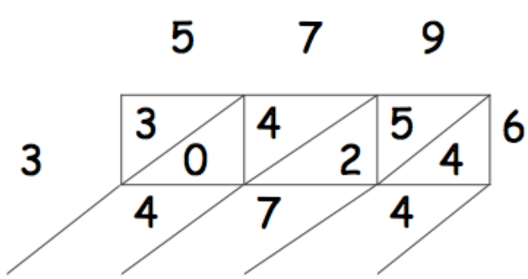
Use the lattice method to multiply 23 x 64

Write 23 above the lattice.
Write 64 on the right side of the lattice.
Multiply 6 x 3. Then multiply 6 x 2.
Multiply 4 x 3. Then multiply 4 x 2.
Write the answers as shown on the lattice.

Add the numbers along each diagonal starting at the right.
For the sum 17, write 7. Then add the 1 to the sum along the diagonal above.

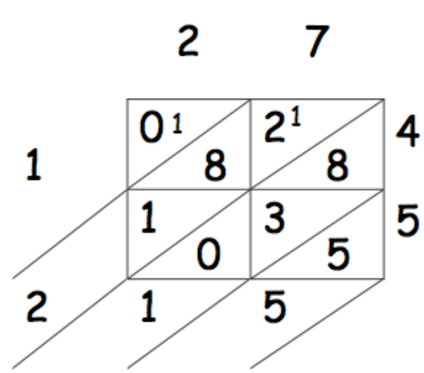
Read the answer: 23 x 64 = 1,472

1. 579 x 6



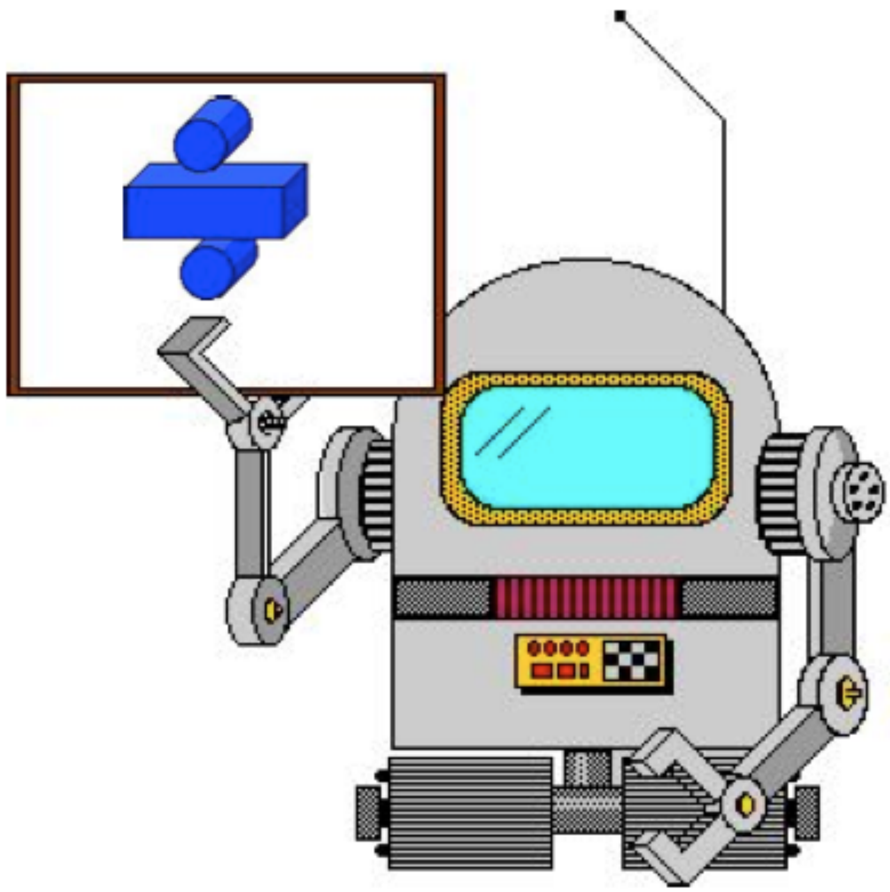
579 x 6 = 3,474

2. 27
x 45



27 x 45 = 1,215

Division Strategies and Algorithms



Repeated Subtraction

What is it?

Students repeatedly subtract the divisor from the dividend until it can no longer be done to produce a positive value. Students then count the number of times the divisor was subtracted, and that is the quotient. The remainder is the difference of the last subtraction problem.

When do you use it?

This strategy can be used to solve division problems with two-digit numbers or basic fact division.

Example: 76 ÷ 15
(Dividend + Divisor)

$$\begin{array}{r} 76 \\ -15 \\ \hline 61 \end{array}$$ (1 time)

$$\begin{array}{r} 61 \\ -15 \\ \hline 46 \end{array}$$ (2 times)

$$\begin{array}{r} 46 \\ -15 \\ \hline 31 \end{array}$$ (3 times)

$$\begin{array}{r} 31 \\ -15 \\ \hline 16 \end{array}$$ (4 times)

$$\begin{array}{r} 16 \\ -15 \\ \hline 1 \end{array}$$ (5 times)

5 remainder 1

Milestones:

If the students can consistently use repeated subtraction, they are ready for equal grouping.

canoes	scouts per canoe	total number of scouts
?	3	24

Equal Grouping

What is it?

In situations where the number in each group and the total number of objects are known, the problem is to find the number of groups.
Equal-grouping problems are also called quotative division.
Many children solve equal-grouping problems by making as many groups of the correct size as possible and then counting the number of groups.

When do you use it?

Students use this strategy with beginning division problems.

Example:

Twenty-four Girl Scouts are going on a canoe trip. Each canoe can hold 3 scouts. How many canoes are needed?

Possible number models:

3x____=24
24/3 = ____
24÷3=____

Milestones:

Students may be using this strategy concurrently with equal sharing and inverse operations.
Students who show understanding of inverse operations and basic multiplication facts are ready to move on to extended division facts.

children	Baseball cards per child	total number of cards
4	?	28

Equal Sharing

What is it?

In situations where the number of groups and the total number of objects are known, the problem is to find the number in each group. Equal Sharing problems are also called partitive division. Many children solve equal-sharing problems by "dealing out" the objects to be shared.

When do you use it?

Students use this strategy with beginning division problems.

Example:

Twenty-eight baseball cards are to be shared equally by 4 children. How many cards does each child get?

Possible number models:

4x____=28
28/4 = ____
28÷4=____

Milestones:

Students may be using this strategy concurrently with equal grouping and inverse operations.
Students who show understanding of inverse operations and basic multiplication facts are ready to move on to extended division facts.

Extended Division Facts

What is it?

Students use basic multiplication facts, and their inverses, to find quotients that are multiples of 10.

When do you use it?

Students use this strategy with division problems involving multiples of 10, 100, etc.

Examples:

280 ÷ 4 = ?
Think: 28÷4=7
Then 280 ÷ 4 is 10 times as much.
280 ÷ 4 = 10 x 7 = 70

16,000 ÷ 8 = ?
Think16÷8=2
Then 16,000 ÷ 8 is 1,000 times as much.
16,000 ÷ 8 = 1,000 x 2 = 2,000

Milestones:

Students who demonstrate fluency with extended facts, and understanding of the place values they are working with, are ready to move on to the Partial-Quotients algorithm.

Partial Quotients

What is it?

Students use several steps to find the quotient by relying on known facts and multiples of 10. The process ends by adding all of the partial quotients together.

When do you use it?

This strategy can be used to solve more complex division problems.

Example: 1,034 ÷ 6

1,034 ÷ 6 = ?

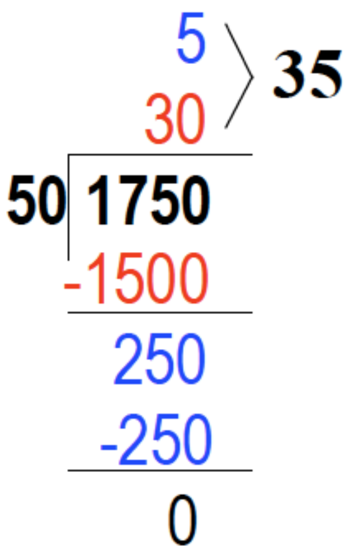
Write partial quotients in the column to the right.

6)1,034	
− 600	100
434	
−300	50
134	
−120	20
14	
−12	2
2	172
Remainder	Quotient

Think: How many [6s] are in 1,034? At least 100
The first partial quotient is 100. There are 100 groups of 6 in 1
Since 100 x 6 = 600, subtract 600 from 1,034.
Think: How many [6s] are left? At least 50
The second partial quotient is 50.
Since 50 x 6 = 300, subtract 300.
Think: How many [6s] are left? At least 20
The third partial quotient is 20.
Since 20 x 6 = 120, subtract 120
Think: What basic fact do I know that is close to 14? 6 x 2
The fourth partial quotient is 2.
Since 6 x 2 = 12, subtract 12.
Add the partial quotients.
The answer is 172 R2

Partial Quotients (Continued)

Example: 1750 ÷ 50



Milestones:

As students become more efficient, and are able to solve problems with partial quotients that are organized by place value, they are ready to move on to the traditional division algorithm.

Division Notes

As students work with partial quotients, they will begin to use larger groups to complete the problems taking them through fewer steps. As they become more efficient with this process they will move into the traditional algorithm.