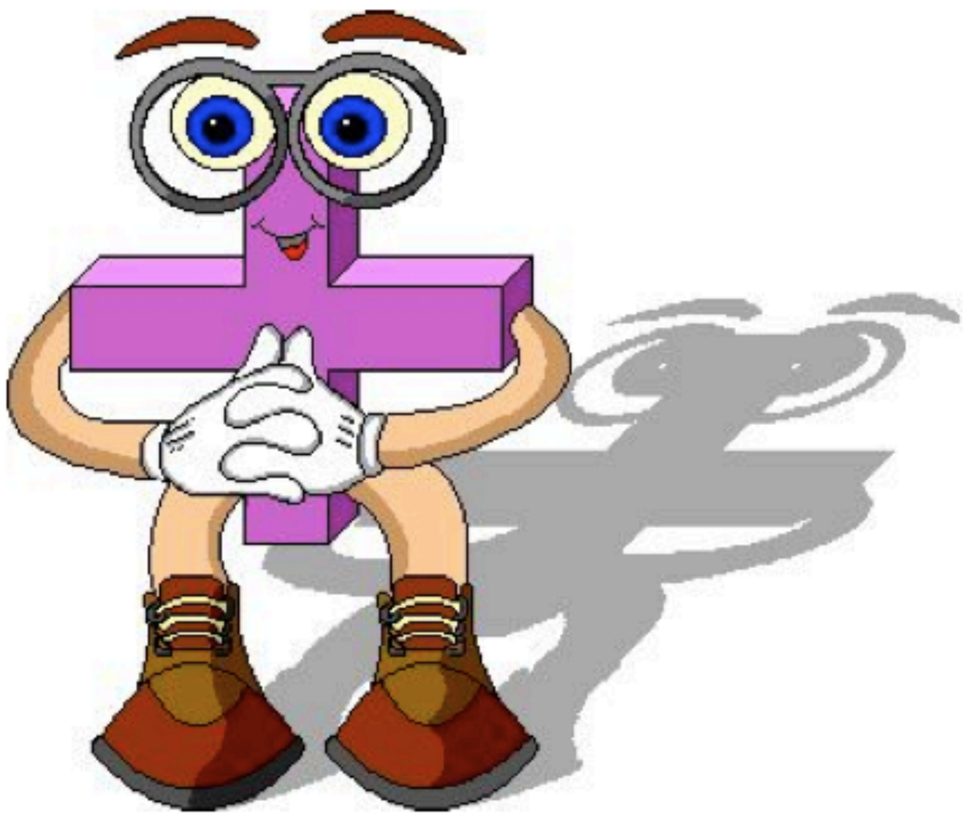
Addition Strategies and Algorithms



Counting All

What is it?

Students use manipulatives to represent addition problems. They will use one-to-one correspondence to count all of the objects in the collection.

When do I use it?

Students use this strategy to find sums of beginning addition problems.

Example:

Students might have an arrangement of manipulatives grouped in a set of 3 and a set of 4. They would count all of the objects to find the sum of 3 + 4.

Milestones:

If students exhibit either of the following characteristics, they are ready to move on to Counting On.
Students begin skip counting when counting objects. (i.e. counting by 2s or 5s instead of by 1s).
Students begin grouping objects and start with the number of the group they've made when they begin counting.

Counting On

What is it?

Students will start with an addend and count on to find the sum.

When do I use it?

Students use this strategy to find sums of beginning addition problems.

Example:

For example, if a student is adding 7 and 3, the teacher would have him: "Grab" 7 (in the air) and say "7." Then, (s)he would count on "8, 9, 10." Students will hold out their fingers as they are counting to represent the 8, 9, and 10.

Milestones:

If students consistently begin with the larger number and view that number as a whole group rather than single items, they are ready to move forward to Making 10 and Shifting. (You will know they are viewing the number as single items if you see them returning to one to begin counting.)

Making 10

What is it?

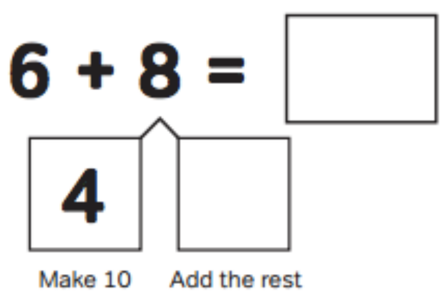
Students are given a number and can state the number needed to make 10.

When do I use it?

Students will use this strategy to find complements of 10.

Example:

For example, if the teacher says 4, the students would respond with “6” because 4 + 6 = 10.



Milestones:

If students can consistently and quickly give the complements of 10 for any given number, they are ready to extend their thinking to making 10s, 100, and 100s, as well as beginning to use partial sums.

Making 10s, 100, and 100s

What is it?

Students are given a number and can state the number needed to make a number such as 20, 30, 70, 100, 400, etc.

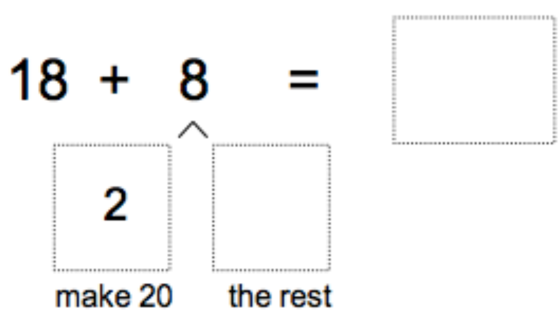
When do I use it?

Students use this strategy when finding complements of multiples of 10.

Example:

For example, if the target is 20 and the teacher says 15, the students would respond with “5” because 15 + 5 = 20
For making 100, students would say 37 if their teacher said 63. 100 is another landmark number students need to learn flexibility with as well.

Example: **18 + 8 = ?**



Milestones:

Look for students to consistently and quickly give the complements of 10 and 10s and develop flexibility with 100 and 100s.
Students may be using this strategy concurrently with partial sums.

Shifting

What is it?

Systemically breaking a number apart.

When do I use it?

Students use this strategy when trying to determine how to break up a number for easier computation.

Example:

For example, if children are using shifting to solve 37 + 56, they may take 4 from the 37 and add it to the 56 to get a friendly number.

Take 4 from 37
And put it with
the 56 to make 60.

$$\cancel{37} \xrightarrow{4} + \ 56$$

Now you have
33 and 60.

$$33 + 60 = 93$$

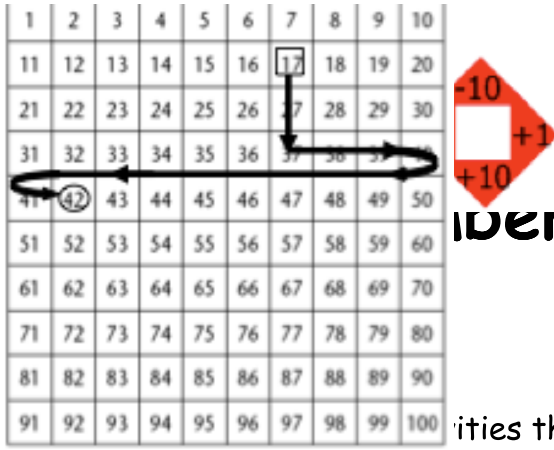
or

37 and 60 is 97.
That's 4 extra,
So it's 93.

$$37 + 60 = 97$$
$$97 - 4 = 93$$

Milestones:

If students can consistently see numbers flexibly and group in ways to make addition easier, they are ready to extend their thinking to partial sums.



ber Grid

What is it?

A number ities that reinforce understanding of numeration and place value. For example, by exploring the patterns in rows and columns, children discover that any number on the number grid is:

When do I use it?

Use the number grid for finding sums of beginning addition problems and two-digit addition problems.

- 1 more than the number to its left
- 1 less than the number to its right
- 10 more than the number above it
- 10 less than the number below it

Example:

17 + 25 = ?

- Start at 17.
- Add 20.
- Move down 2 rows to 37.
- Add 5.
- Count 5 more to 42.

17 + 25 = 42

Milestones:

When students begin adding 10s and 1s in their head (they no longer rely on the number grid) they are ready to move on to Partial Sums.

Partial Sums

What is it?

This algorithm calculates partial sums, working with one place- value column at a time, and then adds all the partial sums to find the total. The partial sums can be found in any order, but working from left to right is often the student preference.

Partial-sums addition is the algorithm most similar to addition with base-10 blocks.

When do I use it?

Students will use this strategy when adding two or more two- or three-digit numbers.

One of the purposes of this algorithm is to help children think flexibly about numbers.

Example:

17 + 25 = ?

10 + 7 20 + 5

- Add the 10s 10 + 20 = 30
- Add the 1s 7 + 5 = 12
- Add the partial sums 30 + 12 = 42

So, 17 + 25 = 42

Milestones:

If the students are increasing their efficiency and not needing every part/step written out for accuracy, they are ready to extend their thinking to partial-partial sums.

Partial-Partial Sums

What is it?

This algorithm is an extension of the Partial Sums Algorithm. However, students are now breaking apart only one of the addends.

When do I use it?

Students will use this strategy when adding two or more two- or three-digit numbers.

Example:

- 17 + 25 = ?
  - Add the 20 to 17. 17 + 20 = 37
  - Add the 5 ones to the previous partial sum 37 + 5 = 42

  So, 17 + 25 = 42

As children grow in their flexibility with numbers, you may begin to see them using the "Make 10" Strategy with Partial Sums.

- 37 + 8 = ?

  3 + 5
  - 37 + 3 = 40
  - 40 + 5 = 45

Milestones:

Some students will be doing this concurrently with column addition. (The Partial-Partial Sums algorithm is more of a mental math strategy.)

Column Addition

What is it?

In the column-addition algorithm, vertical lines are drawn to separate ones, tens, hundreds, and so on. Once columns have been created, the usual place-value convention that each place must have only one digit can be broken without confusion. The digits in each column are then added, beginning in any column. Finally, any necessary trades are made, again starting in any column.

When do I use it?

Students will use this algorithm when adding two- and three- digit numbers.

Example:

Example:

	Tens	Ones
	4	4
+	3	9
	7	13

Step 1: Add the digits in each column

	Tens	Ones
Final Answer	8	3

Step 2: Regroup if necessary

Milestones:

When the students are able to regroup without writing the number in the column, they are ready to move on to the traditional place value algorithm.

Transitioning

What might the transition from Partial Sums to Place Value (or Traditional) look like?

The entire Partials Algorithm is built on the premise of children thinking of numbers in pieces. We begin with children looking at problems horizontally so that they are more concerned with decomposing the number into its pieces than a procedure for solving the problem.

Once children understand the process behind adding numbers in parts, it is time to transition them from horizontal problems to vertical problems.

Beginning Understanding

27 + 15 = ?

20 + 10 = 30
7 + 5 = 12
30 + 12 = 42

Intermediate Understanding

$$\begin{array}{r} 27 \\ \underline{+15} \\ 30 \\ \underline{+12} \\ 42 \end{array}$$

Advanced Understanding

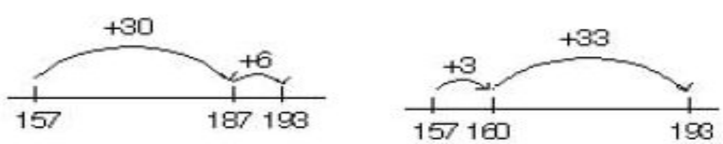
$$\begin{array}{r} \overset{1}{2}7 \\ \underline{+15} \\ 42 \end{array}$$

Beginning preliminary discussions about what the 1 in 12 means will lend itself to the transition to the traditional algorithm.

Open Number Lines

The open number line allows students to see the variety of ways that the same question can be solved.

For example, to solve 157 + 36 one student may begin at 157, add 30, then 6 while another may start at 157 and break the 36 into 3 and 33. This turns the question into the problem of adding 33 to 160.



Writing equations horizontally forces students to look at and think about the numerals, whereas written vertically students tend to immediately turn to the procedures, often abandoning reason.

Subtraction Strategies and Algorithms

Counting Back/Counting Up

What is it?:

Students will start with the biggest number and count back to find the difference.

OR

Students will start with the number that is being subtracted and count up until they reach the biggest number.

When do I use it?

Students used this strategy for finding differences of beginning subtraction problems

Example:

For example, if a student is subtracting 3 from 10, the teacher would have him:

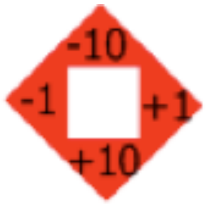
Say "10." Then, he would count back, "9, 8, 7." Student will hold out his fingers as he is counting to represent the 3 that are taken away from 10.

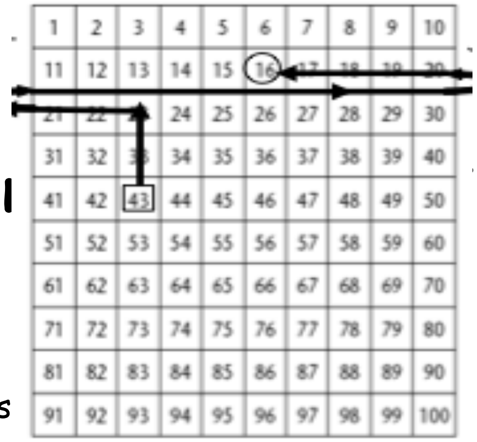
OR

Say "3". Then he would count on "4, 5, 6, 7, 8, 9, 10". The student will hold out a finger for each number he is counting up and then count the fingers to represent the answer (7)

Milestones:

If students consistently begin with the larger number and view that number as a whole group rather than single items, they are ready to move forward. (You will know they are viewing the number as single items if you see them returning to one to begin counting.)





Co[illegible] [illegible]k o[illegible] d

What is it?

Students start at the bigges

When do I use it?

Students use this strategy for subtracting two digit numbers.

Example:

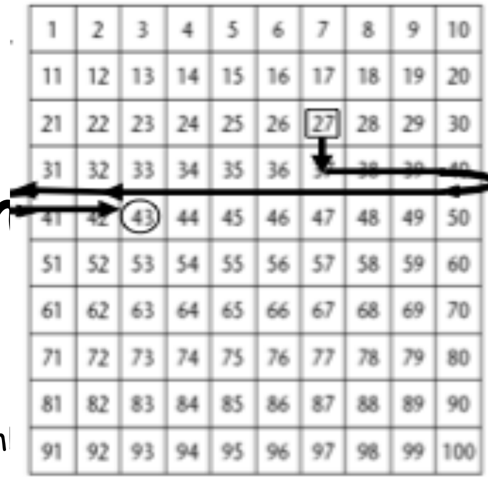
43 – 27 = ?

- Start at 43
- Subtract 20
- Move up 2 rows to 23
- Subtract 7
- Count back 7 to 16

43 – 27 = 16

Milestones:

Look for children to begin to do this process mentally without the aid of a number grid.



Count Up or d

What is it?

Students start with the num on.

When do I use it?

Students use this strategy for subtracting two digit numbers.

Example:

43 – 27 = ?

- Count up 10
- Move down 1 row to 37
- Count up 6 more.
- Count up 6 more to 43

43 – 27 = 16

Milestones:

Look for children to begin to do this process mentally without the aid of a number grid.

Counting-Up Subtraction Method

What is it?

Starting at the smaller number, count up to make the nearest ten, count on by tens and hundreds, and then count up by ones. Finally, add all of the numbers you added on to find the difference.

When do I use it?

Students use this strategy when subtracting two digit (or larger) numbers.

Example:

Subtract 89 from 347 by counting up.

Write the smaller number and count up to the larger number.
Each time you count up, circle that number.

89	
+ 1	Count up to the nearest 10.
90	
+ 10	Count up to the nearest 100.
100	
+ 200	Count up to the largest possible hundred.
300	
+ 47	Count up to the larger number.
347	

Next, add the numbers you circled. 1 +10 +200 +47 = 258
You counted up 308. Therefore. 347-89 = 258

Milestones:

Look for students to begin to do this mentally, and to cluster their addition into fewer groups.

Same-Change

What is it?

If you add the same number to both numbers in the problem, your new subtraction problem will have the same answer as the original problem. If you subtract the same number from both numbers in the problem, your new subtraction problem will have the same answer as the original problem.

When do I use it?

Students use this algorithm for subtracting two-digit (or larger) numbers.

Example:

Example: 83 – 27 = ?	(Add 3)	86
**No Trading Necessary Now		– 30
		56
Example: 500 – 257 = ?	(Subtract 1)	499
**No Trading Necessary Now		– 256
		243

Milestones:

As students gain understanding of equivalence, they will be able to mentally make changes to more efficiently compute differences.

Partial-Differences

What is it?

In this method the children subtract from left to right, one column at a time.
In some cases, the larger number is on the bottom and the smaller number is on the top. When you subtract these numbers, the difference will be a negative number.

When do I use it?

Students use this algorithm for subtracting two digit (or larger) numbers. (Proceed with caution and watch for misconceptions.)

Example:

835 - 472 = ?

		835
		-472
Subtract the 100s	800 - 400	400
Subtract the 10s	30 - 70	- 40
Subtract the 1s	5 - 2	3
Find the total.	400 - 40 + 3	363

When subtracting 70 from 30, think of first subtracting 30 and then subtracting 40 more.

30 - 30 - 40 = -40

835 - 472 = 363

Milestones:

This algorithm may be used in place of or concurrently with the traditional subtraction algorithm.
Students who have a strong number sense will quickly understand that subtracting a larger number from a smaller number will result in a negative difference.
Students who have a strong understanding of subtraction will be able to subtract the larger number from the smaller number regardless of its position in the problem as well as understand whether the answer is positive or negative.

Trade-First Subtraction Method

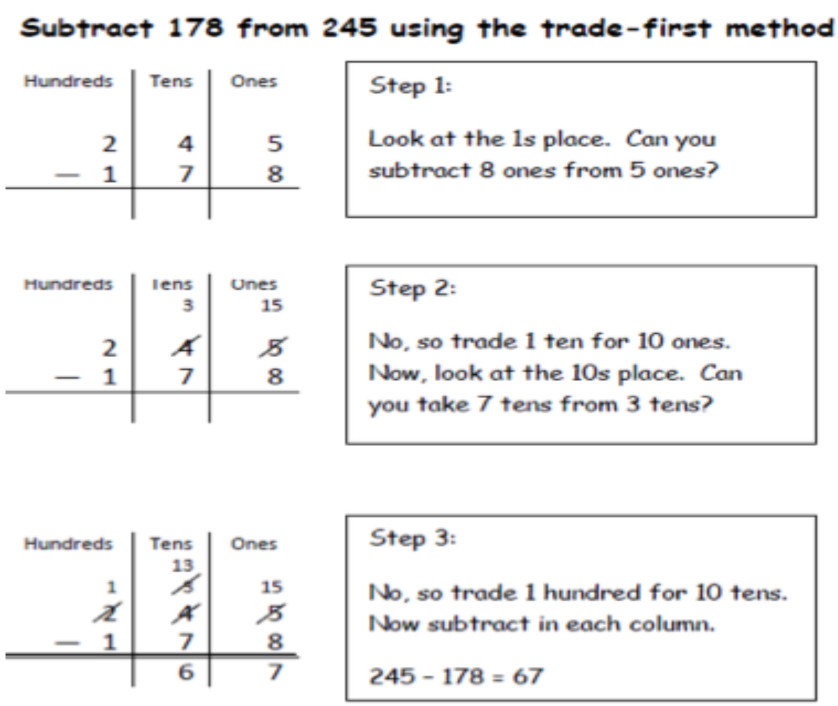
What is it?

In this method the children look at all of the numbers first and determine if they need to trade before subtracting.

When do I use it?

Students use this algorithm for subtracting two digit (or larger) numbers.

Example:



Milestones:

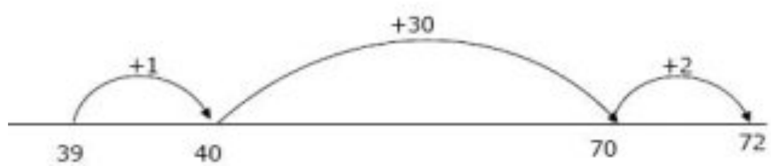
This algorithm may be used in place of or concurrently with the traditional subtraction algorithm.

The only difference between this algorithm and the traditional algorithm is the order in which you trade and subtract.

Open Number Lines

Subtracting 2 digits from 2 digits:

I need 72 dollars to buy a skateboard. I have 39 dollars already. How many more dollars do I need to save?



Place 39 near the start of the open number line and 72 near the end. We can count up in 'friendly' jumps to reach 72. First a jump of 1 to reach 40 (multiples of ten are easy numbers to jump to and from), then a jump of 30 to reach 70 and finally a jump of 2 to reach our target of 72. I need to save 33 more dollars.